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1999 J. Phys.: Condens. Matter 11 4341

(http://iopscience.iop.org/0953-8984/11/22/306)

J. Phys.: Condens. Matter **11** (1999) 4341–4347. Printed in the UK PII: S0953-8984(99)98949-X

Cellular effective-medium approximation for the giant magnetoresistance in magnetic granular films

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Received 4 November 1998, in final form 8 February 1999

Abstract. To derive the conductivity and magnetoresistance of inhomogeneous magnetic granular films, a cellular effective-medium approximation is developed to include spin-dependent interfacial scattering and bulk scattering. It is shown that the giant magnetoresistance as well as the conductivities of these systems depend strongly on the volume fraction and the size of the magnetic granules.

1. Introduction

Giant magnetoresistance (GMR) is not exclusive to multilayer geometries [1–5]. It has recently been found that GMR can also be achieved in magnetic granular films [6–12]. It is believed that the main mechanism giving rise to the GMR in these system is the spin-dependent scattering of the conduction electrons by magnetic impurities. The magnetoresistance (MR) in magnetic granular films is associated with a change in the relationship of the orientations of the magnetizations in neighbouring particles.

Recently, some authors [13–16] have developed various models of GMR in granular magnetic materials in order to discuss the influence of the magnetic particle size and electron free path on the GMR. But they have not discussed the relation between the volume fraction of the magnetic particles and the GMR. Now we extend the cellular effective-medium approximation (CEMA) [1] to study the transport properties in magnetic granular films and discuss the dependence of the GMR on the volume fraction of the magnetic particles. We focus on the case where the spin diffusion length is much larger than both the effective mean free path of the system and the size of the ferromagnetic regions, so that the spin-flip effect can be neglected and the two-current model is applicable.

2. Formalism

2.1. The general expression for conductivity in the CEMA

We first consider a simple model of a heterogeneous medium. Let the medium consist of granules with conductivity σ_1 and σ_2, present in volume fractions f and $1 - f$ (see figure 1). The effective conductivity σ^* is defined by the equation

$$\langle J \rangle = \sigma^* \langle E \rangle \tag{1}$$

where $\langle J \rangle$ and $\langle E \rangle$ denote spatial averages of the current density and the field. We consider two types of spherical cell, which have the same radius R. As shown in figure 1, the first cell, A, consists of a core with conductivity σ_1 and a shell with conductivity σ_2; the conductivity of the second cell, B, is σ_2. The material, or the volume V, is subdivided into these cells without overlapping [17].

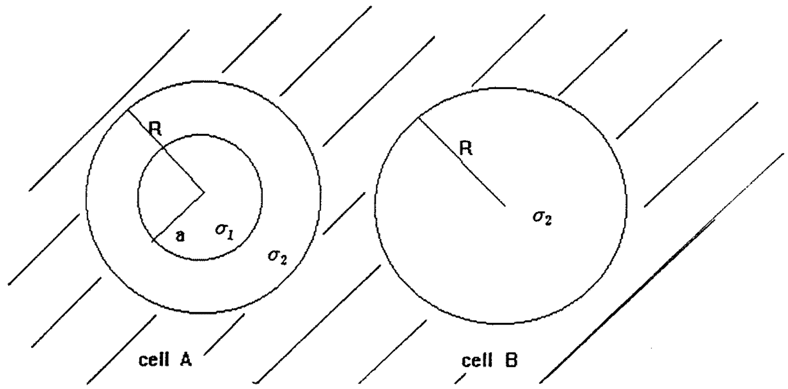

Figure 1. The model of the two-phase material. Different types of cell are embedded in the effective medium having conductivity σ^*.

First, we take cell A into account. It is shown in reference [18] that a coated grain can be regarded as an equivalent solid grain. On the basis of the same idea, we can obtain an expression for the conductivity σ_A of the cell A:

$$\sigma_A = \mu\sigma_1 \tag{2}$$

$$\mu = \frac{\gamma(1+2\gamma) + 2\lambda\gamma(1-\gamma)}{(1+2\gamma) - \lambda(1-\gamma)} \tag{3}$$

$$f_A = \frac{f}{\lambda} \tag{4}$$

with $\gamma = \sigma_2/\sigma_1$, $\lambda = (a/R)^3$. Here a is the radius of the core. According to the CEMA [17], the relation between λ and f is

$$\lambda = f + 8f^2 - 12f^3. \tag{5}$$

Now we consider cell B; the conductivity of cell B is

$$\sigma_B = \sigma_2 \tag{6}$$

and the volume fraction $f_B = 1 - f_A$. The field and current in a cell of the ith species are uniform and are given by

$$\vec{E}_{\mathrm{in},i} = \frac{3\sigma^*}{2\sigma^* + \sigma_i}\vec{E}_{\mathrm{far}} \tag{7}$$

$$\vec{J}_{\mathrm{in},i} = \sigma_i \vec{E}_{\mathrm{in},i}. \tag{8}$$

Here i = A or B, The averages $\langle J \rangle$ and $\langle E \rangle$ take the forms

$$\langle \vec{E} \rangle = \sum_i f_i \vec{E}_{\mathrm{in},i} \tag{9}$$

$$\langle \vec{J} \rangle = \sum_i f_i \vec{J}_{\mathrm{in},i} \tag{10}$$

and the self-consistency condition for a two-component composite [17] is

$$f_A \frac{\sigma^* - \sigma_A}{2\sigma^* + \sigma_A} + f_B \frac{\sigma^* - \sigma_B}{2\sigma^* + \sigma_B} = 0. \tag{11}$$

Thus, we get

$$\sigma^* = \frac{1}{4}[(3f_B - 1)\sigma_B + (3f_A - 1)\sigma_A] + [(3f_B - 1)\sigma_B + (3f_A - 1)\sigma_A]^2 + 8\sigma_A\sigma_B]^{1/2}. \tag{12}$$

2.2. The CEMA expression for the effective conductivity of the ferromagnetic coated granular system

Now we consider a system composed of parallel ferromagnetic coated granules with conductivity σ_α^a (the core with conductivity σ_α^f and the shell with conductivity σ_α^m) and non-magnetic granules with conductivity σ_α^n. Here $\alpha = +\ (-)$ refers to the majority- (minority-) spin direction. In the absence of the magnetic field, the magnetizations of the granules are random. In most of the theoretical work on GMR, only collinear magnetization configurations of the granules are considered, e.g. all the ferromagnetic granules are assumed to have only two magnetization directions: up (↑) and down (↓), with the quantization axis along the direction of the applied magnetic field. Such a simplified treatment is an approximation of a magnetic granular system, which is used in the Gu *et al* model [15]. In the present model we also use this collinear approximation and expect the results obtained to be valid at least qualitatively. To obtain the effective conductivity of the system. We first view the coated magnetic particle (with conductivity $\sigma_\alpha^f, \sigma_\alpha^m$, volume fraction f) as a solid grain (with conductivity σ_α^a, volume fraction f) [18]. Then, through the CEMA, the effective conductivities $\sigma_\alpha^M, \sigma_\alpha^D$ (here the σ_α^M ($\alpha = +, -$) are the average conductivities for spin-up and spin-down channels in the completely magnetized state; the σ_α^D ($\alpha = +, -$) are the average conductivities for spin-up and spin-down channels in the demagnetized state) are obtained through the self-consistent equation for σ_α^A, f_A, σ_α^B, f_B.

It is found that the expression for the conductivity of a concentrically coated magnetic particle σ_α^a is

$$\sigma_\alpha^a = \mu_\alpha \sigma_\alpha^f \tag{13}$$

where

$$\mu_\alpha = \frac{\gamma_\alpha(1 + 2\gamma_\alpha) + 2\lambda\gamma_\alpha(1 - \gamma_\alpha)}{(1 + 2\gamma_\alpha) - \lambda(1 - \gamma_\alpha)} \tag{14}$$

with

$$\gamma_\alpha = \sigma_\alpha^m / \sigma_\alpha^f \qquad \lambda = \left(\frac{a}{a+t}\right)^3.$$

Here a and t are the radius of the core and the thickness of the shell respectively. Taking the limit $t \to 0$, the ratio $r_\alpha = t/\sigma_\alpha^m$ has a finite value and μ_α is reduced to

$$\mu_\alpha = a/(a + r_\alpha \sigma_\alpha^f).$$

Now, we let $\sigma_\alpha^a = \sigma_1, \sigma^n = \sigma_2$. Equation (11) changes into

$$f_A \frac{\sigma_\alpha^* - \sigma_\alpha^A}{2\sigma_\alpha^* + \sigma_\alpha^A} + f_B \frac{\sigma_\alpha^* - \sigma^B}{2\sigma_\alpha^* + \sigma^B} = 0 \tag{15}$$

where

$$\mu_\alpha = \frac{\gamma_\alpha(1+2\gamma_\alpha) + 2\lambda\gamma_\alpha(1-\gamma_\alpha)}{(1+2\gamma_\alpha) - \lambda(1-\gamma_\alpha)} \tag{16}$$

$$\sigma^{\mathrm{B}} = \sigma^{\mathrm{n}} \tag{17}$$

$$\gamma_\alpha = \sigma^{\mathrm{n}}/\sigma_\alpha^{\mathrm{a}} \tag{18}$$

$$f_{\mathrm{A}} = f/\lambda \tag{19}$$

$$\lambda = \left(\frac{a}{R}\right)^3 \tag{20}$$

$$f_{\mathrm{B}} = 1 - f_{\mathrm{A}} \tag{21}$$

$$\lambda \approx f + 8f^2 - 12f^3. \tag{22}$$

Here f is the volume fraction of the core. From equation (15) we can get the effective conductivity of this system, σ_α^*.

MR is defined as the difference in resistivity between the completely magnetized state (ρ^{M}) and the completely demagnetized state (ρ^{D}). For the magnetized state, all of the magnetic granules have the same magnetization direction (pointing up); the effective conductivity σ^{M} is

$$\sigma^{\mathrm{M}} = \sigma_+^{\mathrm{M}} + \sigma_-^{\mathrm{M}} \tag{23}$$

and σ_+^{M}, σ_-^{M} satisfy the self-consistency conditions

$$f_{\mathrm{A}} \frac{\sigma_+^{\mathrm{M}} - \sigma_+^{\mathrm{A}}}{2\sigma_+^{\mathrm{M}} + \sigma_+^{\mathrm{A}}} + f_{\mathrm{B}} \frac{\sigma_+^{\mathrm{M}} - \sigma^{\mathrm{B}}}{2\sigma_+^{\mathrm{M}} + \sigma^{\mathrm{B}}} = 0 \tag{24}$$

$$f_{\mathrm{A}} \frac{\sigma_-^{\mathrm{M}} - \sigma_-^{\mathrm{A}}}{2\sigma_-^{\mathrm{M}} + \sigma_-^{\mathrm{A}}} + f_{\mathrm{B}} \frac{\sigma_-^{\mathrm{M}} + \sigma^{\mathrm{B}}}{2\sigma_-^{\mathrm{M}} + \sigma^{\mathrm{B}}} = 0. \tag{25}$$

For the demagnetized state, there are equal numbers of magnetic granules with magnetization pointing up and down, so the effective conductivity σ^{D} is

$$\sigma^{\mathrm{D}} = \sigma_+^{\mathrm{D}} + \sigma_-^{\mathrm{D}} = 2\sigma_+^{\mathrm{D}} \tag{26}$$

and σ_+^{D}, σ_-^{D} satisfy the self-consistency condition

$$\frac{f_{\mathrm{A}}}{2}\left(\frac{\sigma_+^{\mathrm{D}} - \sigma_+^{\mathrm{A}}}{2\sigma_+^{\mathrm{D}} + \sigma_+^{\mathrm{A}}} + \frac{\sigma_-^{\mathrm{D}} - \sigma_-^{\mathrm{A}}}{2\sigma_-^{\mathrm{D}} + \sigma_-^{\mathrm{A}}}\right) + \frac{f_{\mathrm{B}}}{2} \frac{\sigma_+^{\mathrm{D}} - \sigma^{\mathrm{B}}}{2\sigma_+^{\mathrm{D}} + \sigma^{\mathrm{B}}} = 0. \tag{27}$$

Then the MR of the system is obtained from

$$\mathrm{MR} = \Delta\rho/\rho^{\mathrm{D}} = \Delta\sigma/\sigma^{\mathrm{M}} = (\sigma^{\mathrm{M}} - \sigma^{\mathrm{D}})/\sigma^{\mathrm{M}}$$

with $\sigma^{\mathrm{M}} = 1/\rho^{\mathrm{M}}$, $\sigma^{\mathrm{D}} = 1/\rho^{\mathrm{D}}$.

3. Discussion

We have extended the CEMA to study the GMR effect in an inhomogeneous magnetic granular system. We here assume that the spin-asymmetric factors in the cores and in the shells are $n_1 = \sigma_+^{\mathrm{f}}/\sigma_-^{\mathrm{f}}$, $n_2 = \sigma_+^{\mathrm{m}}/\sigma_-^{\mathrm{m}}$, respectively. And we let $\sigma^{\mathrm{n}} = \sigma_-^{\mathrm{f}}$; then we get the expression for the MR.

Figure 2 shows the volume-fraction dependence of the GMR. It is shown that for a low volume fraction of magnetic particles, the GMR is small and increases with the volume-fraction increase. In the middle region, the MR reaches a maximum with the largest GMR effect. As the volume fraction increases further, the MR decreases. This is because for small values of the volume fraction, the ferromagnetic particles are few and far apart, resulting in fewer magnetic

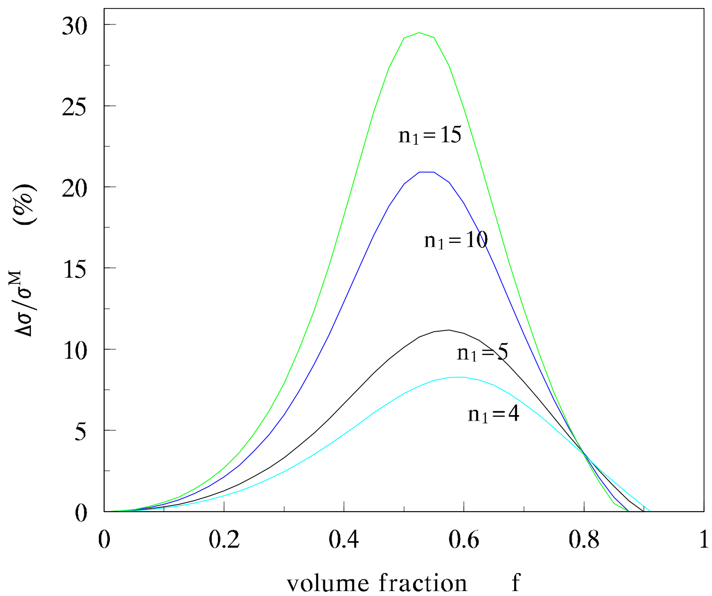

Figure 2. The volume-fraction dependence of the GMR for different spin-asymmetric factors $n_1 = \sigma_+^f/\sigma_-^f$. ($n_2 = \sigma_+^m/\sigma_-^m = 36$, $a = 5r_-\sigma_-^f$.) Here n_1, n_2 are the spin-asymmetric factors in the core and in the shell. $\sigma_\alpha^f, \sigma_\alpha^m$ are the core conductivity and the shell conductivity where $\alpha = +, -$; $r_- = t/\sigma_-^m$; a and t are the radius of the core and the thickness of the shell.

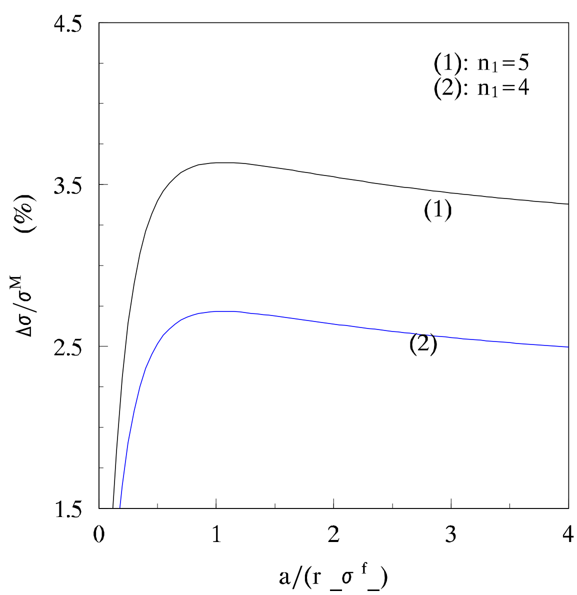

Figure 3. The percentage of GMR ($\Delta\sigma/\sigma^M$) as a function of the particle size ($a/r_-\sigma_-^f$) for systems of spherical granules. ($n_1 = \sigma_+^f/\sigma_-^f$, $n_2 = \sigma_+^m/\sigma_-^m = 36$, $f_1 = 0.35$.) Here n_1, n_2 are the spin-asymmetric factors in the core and in the shell. $\sigma_\alpha^f, \sigma_\alpha^m$ are the core conductivity and the shell conductivity where $\alpha = +, -$; $r_- = t/\sigma_-^m$; a and t are the radius of the core and the thickness of the shell.

scattering events and hence a small MR. As the volume fraction is large, the magnetic particles begin to consolidate into large ones; the surface–volume ratio of the particles decreases which reduces the spin-dependent scattering, then the MR decreases. Between these limits lies the maximal GMR. This behaviour is in agreement with experimental results [10].

Figure 3 shows the MR as a function of magnetic particle size. There is always an optimum particle size for the GMR. In view of the fact that in most experiments [6–12] a maximum of GMR is observed as a function of the annealing temperature and therefore as a function of the particle size, our results are in agreement with experimental observations [6–12].

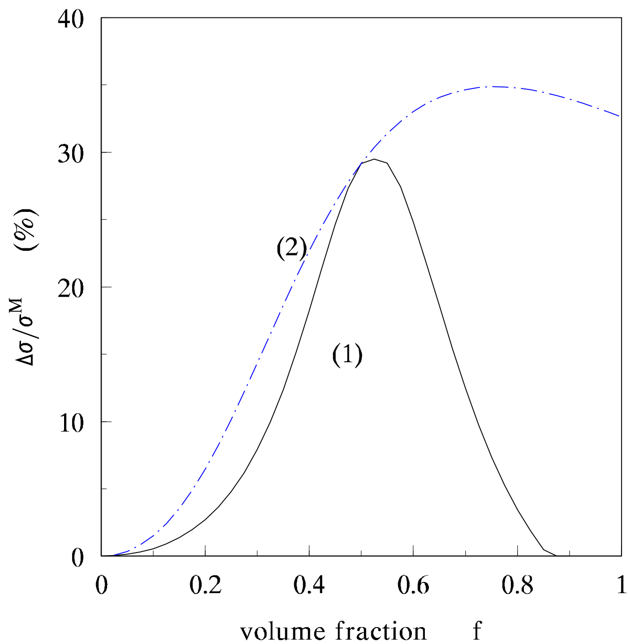

Figure 4. The percentage of GMR ($\Delta\sigma/\sigma^{\mathrm{M}}$) as a function of the volume fraction of the ferromagnetic granules. The results come from the CEMA (curve 1) and the EMA (curve 2). ($n_1 = \sigma_+^{\mathrm{f}}/\sigma_-^{\mathrm{f}} = 15, n_2 = \sigma_-^{\mathrm{m}} = 36, a/r_- a_-^{\mathrm{f}} = 5$.) Here n_1, n_2 are the spin-asymmetric factors in the core and in the shell. $\sigma_\alpha^{\mathrm{f}}, \sigma_\alpha^{\mathrm{m}}$ are the core conductivity and the shell conductivity where $\alpha = +, -$; $r_- = t/\sigma_-^{\mathrm{m}}$; a and t are the radius of the core and the thickness of the shell.

Finally, in figure 4, we compare our results for the dependence of the GMR on the volume fraction of the magnetic particles with the results from the effective-medium approximation (EMA) [19]. The EMA due originally to Bruggeman [20] has been widely applied to transport phenomena in inhomogeneous systems. In the EMA, each inhomogeneity is assumed to be embedded in some 'effective medium' that is to be determined self-consistently. From figure 4, one can see that the CEMA results obtained here are closer to the experimental observations.

Acknowledgments

We would like to thank Professor Zhen-ya Li and Professor Ding-yu Xing for helpful discussions. This work was partially supported by the Natural Science Foundation of China.

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